General Relativity - PHY 6938

HW 9

Hand in this homework.

READ: Chap 7.1-7.7

PROBLEMS:

1. Harmonic gauge:
a) Rewrite $\nabla_\mu \nabla^\mu f$ in terms of the determinant of the metric and partial derivatives.
b) Use your result from a) for the function $f = x^\alpha$ to find an expression for $\nabla_\mu \nabla^\mu x^\alpha$.
c) Show that the condition $\nabla_\mu \nabla^\mu x^\alpha = 0$ for harmonic gauge is equivalent to the Lorenz gauge if the metric is of the form $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $h_{\mu\nu}$ is a perturbation.

2. Consider two orbiting particles of masses m_1 and m_2.
a) Derive an expression for $\bar{h}_{ij}$ assuming the particles move on Newtonian circular orbits with separation r.
b) Compute the power P of the gravitational radiation for Newtonian circular orbits.
c) Write down the total Newtonian energy E of the system in terms of the orbital angular velocity ω.
d) Use $\frac{dE}{dt} = -|P|$ to figure out ω changes over time due to the emission of gravitational radiation. Find $\omega(t)$ and also $r(t)$. [Hint: We assume that the orbit remains "circular" but with a slowly changing radius.]
e) Use your results from a) and d) to derive an expression for $\bar{h}_{ij}(t)$ in terms of $\omega(t)$.
f) Sketch how h_+ changes over time.

Do the following problems from Carroll's book: 7.7

If you still have time, also look at 7.9 from Carroll's book.

Note about 7.9:
Carroll's expression (7.165) seems to have typos: $h_{\mu\nu}$ should be replaced by $\bar{h}_{\mu\nu}$. Then we need to do a gauge transformation:

$$\bar{h}_{\mu\nu} \to \bar{h}^{new}_{\mu\nu} + 2\partial_{(\mu}\xi_{\nu)} - \eta_{\mu\nu}\partial_\rho \xi^\rho$$

Schematically $t_{\mu\nu}$ will change like this

$$t \sim \partial h \partial h \to t \sim \partial h \partial h \;\;\&\;\; \partial\xi\partial h \;\;\&\;\; \partial\xi\partial\xi$$

The terms $\partial h \partial h$ are what we want. Using integration by parts (under the average integral) the terms $\partial\xi\partial h$ can be rearranged into the schematic form $\partial\xi\partial h \sim \xi\partial\partial h$. Using the Einstein eqn at linear order one should be able to show that the $\partial\partial h$ terms add up to zero. Using integration by parts the $\partial\xi\partial\xi$ terms should also add up to zero.