Mechanics - PHY 6247

HW 9

READ: Chap. 10: p. 430-477

HOMEWORK:

1. a) Obtain Hamilton's principal function $S(z,t) = W(z,t) - Et$, for a particle of mass m and total energy E which moves vertically in the uniform gravitational field g near the surface of the earth, by integrating the time-independent Hamilton-Jacobi equation.
b) Use $S(z,t)$ to generate the general solution to the dynamical problem: $(z(t), p(t))$

2. The motion of a projectile near the surface of the earth is governed by the Hamiltonian $H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) + mgy$ where x and y are the horizontal and vertical coordinates and p_x and p_y their corresponding conjugate momenta.
a) Find W, the solution to the time-independent Hamilton-Jacobi equation (Hint: use separation of variables)
b) Use your solution to obtain x and y as functions of time.

3. Consider a particle of mass m moving in a 3D isotropic harmonic oscillator potential $V = \frac{m}{2}\omega^2(x^2 + y^2 + z^2)$
a) Separate the Hamilton-Jacobi equation in Cartesian coordinates (x, y, z), find the action variables, and express the Hamilton in terms of these. Find the frequencies $(\omega_x, \omega_y, \omega_z)$.
b) Separate the Hamilton-Jacobi equation in cylindrical coordinates (ρ, ϕ, z), find the action variables, and express the Hamilton in terms of these. Find the frequencies $(\omega_\rho, \omega_\phi, \omega_z)$.

4. Consider a particle with charge q in a homogeneous magnetic field of strength B. The Lagrangian in cylindrical coordinates is

$$L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) + \frac{q}{2}Br^2\dot{\theta}.$$

a) Convince yourself that the particle will always move along a helix with constant radius.
b) Find $\dot{\theta}$ in a coordinate system where r is the constant helix radius.
c) Find the Hamiltonian H in cylindrical coordinates.
d) Show that H is completely separable, and find W_θ and W_z. Find W_r in terms of an integral (that you do not need to solve explicitly).
e) Assume that the particle is on a helical orbit with radius r_0, and velocities $\dot{\theta}_0$, $\dot{z}_0$. How will the radius and the velocities change if B is very slowly increased by a factor of 4?