Mechanics - PHY 6247

HW 7

READ: Chap. 8: p. 334-362 , Chap. 9: p. 368-408

HOMEWORK:

1. Show that the symplectic condition in the form $MJ = J(M^T)^{-1}$ is equivalent Eqns. (9.48a) and (9.48b) in Chapter 9 of the book.

2. Prove Jacobi's identity for Poisson brackets.

3. Determine whether the transformation
$Q_1 = q_1 q_2, \quad P_1 = \frac{p_1 - p_2}{q_2 - q_1} + 1$
$Q_2 = q_1 + q_2, \quad P_2 = \frac{q_2 p_2 - q_1 p_1}{q_2 - q_1} - (q_2 + q_1)$
is canonical.

4. The motion of a particle of mass m undergoing constant acceleration a in one dimension is described by $x = x_0 + \frac{p_0}{m}t + \frac{a}{2}t^2, \;\; p = p_0 + mat$
Consider now a transformation from (x, p) to (x_0, p_0). Is this transformation canonical?

5. Consider now the transformation
$Q = -p, \quad P = q + Ap^2.$
a) Is this transformation canonical?
b) Using these new variables (Q, P), find the new Hamiltonian for a particle moving vertically in a uniform gravitational field g given by $H = \frac{p^2}{2m} + mgq$. Show that we can make Q cyclic by choosing the constant A appropriately.
c) Using this new Hamiltonian, solve the equations of motion and then use the transformation equations to find the original variables (q, p) as functions of time.