Mechanics - PHY 6247

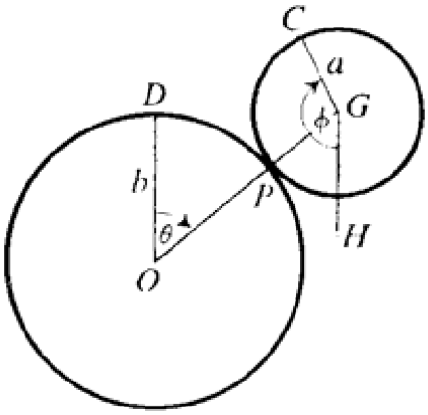
HW 5

READ: Chap. 5: p. 185-232

HOMEWORK:

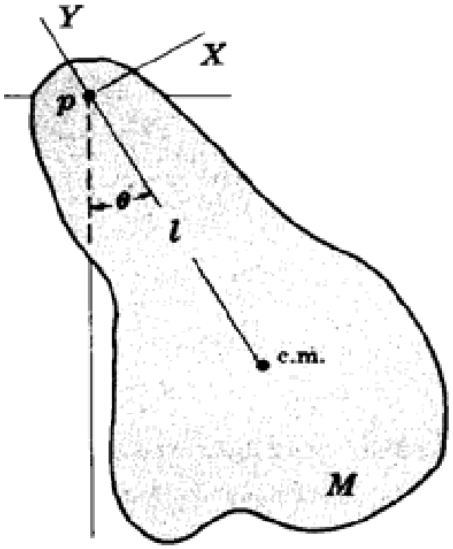
1. Let M be a real, symmetric square matrix.
a. Show that all its eigenvalues are real.
b. Show that there is an orthogonal square matrix D (i.e. a coordinate transformation) such that $D^T M D$ is a diagonal matrix.

2. Under the influence of gravity a cylinder of uniform density with radius a and mass M is rolling without slipping over a second fixed cylinder of radius b. The axes of both cylinders are parallel at all times.
a. Show that the angular velocity of the mobile cylinder is $\frac{a+b}{a}\dot{\theta}$
b. Write down the kinetic and potential energy of this cylinder.
c. Derive the equations of motion using the Lagrangian formalism.



3. A uniform cone of height h, half angle α and density ρ rolls on its side without slipping on a uniform horizontal plane in such manner that it returns to its original position in a time τ. Find expressions for the kinetic and potential energy. Use the Lagrangian formalism to derive the equations of motion.

4. A physical pendulum like the one in the figure consists of an object pivoted at p and free to swing in a vertical plane through an angle θ. Find the equation of motion as a function of the components of the inertia tensor I_{jk} (assumed known), the mass M and the length to the CM l.



5. Same as Problem 4, but with the pendulum attached to a massless rod as in the figure. Keep in mind that this pendulum is confined to move on the vertical plane.
a) Find kinetic and potential energy.
b) Find the equations of motion from the Lagrangian