Mechanics - PHY 6247

HW 3

READ: Chap. 3: p. 70-126

HOMEWORK:

1. A particle of mass 1 kg is under the influence of a central force that can be derived from the potential $V(r) = k \ln(r/L)$, where k=900 J and L=1 m. If the angular momentum of the particle is l= 1 kg m^2/s:
a. Plot $V(r)$, $l^2/(2mr^2)$, and the sum of both.
b. Find the radius and energy of the circular orbit.
c. If the particle has now E = -2000 J, find the apsidal points.

2. A planet of mass M is in an elliptic orbit with semimajor axis a and eccentricity $e = 1-\alpha$, where $\alpha \ll 1$ about the Sun. Assume the motion of the Sun can be neglected and that only gravitational forces act. When the planet is at its greatest distance from the Sun (apohelium) it is struck by a comet of mass m, where $m \ll M$, traveling tangentially to the planets orbit and in the same direction. Assuming a completely inelastic collision, find the minimum energy the comet must have to change the new orbit to a parabola. Express your final answer in terms of the given quantities M, a, α, m, and the constant k in the gravitational potential.

3. A particle moves in a force field described by Yukawas potential (widely used in Nuclear Physics)

$$V(r) = -\frac{k}{r}\exp(-\frac{r}{a})$$

where k and a are positive.
a. Write the equations of motion and reduce them to the equivalent one-dimensional problem. Use the effective potential to discuss the qualitative nature of the orbits for different values of energy and angular momentum.
b. Show that if the orbit is nearly circular and if $r_0/a \ll 1$, the apsidal points will advance by an amount proportional to $(r_0/a)^2$ per revolution. Find the advance per revolution. Here r_0 is the radius of the circular orbit.
HINT for b: (i) use $u = 1/r$ and show that $\frac{d^2u}{d\theta^2} + u = F(u) := -\frac{m}{l^2}\frac{dV}{du}$
(ii) Taylor expand $u = u_0 + \delta u$ and $F(u) = F(u_0) + F'(u_0)\delta u$, where u_0 is the u for a circular orbit.
(iii) Find EOM for δu, and show that solution is $\delta u = A\cos\beta(\theta-\theta_0)$. (What is β equal to?)
(iv) Look at $\beta\Delta\theta = 2\pi$.
(v) In order to use $r_0/a \ll 1$ we need to get rid of the exponential $\exp(-\frac{r_0}{a})$ in $F'(u_0)$. So we rewrite the $\exp(-\frac{r_0}{a})$ in $F'(u_0)$ in terms of u_0 using $F(u_0) = u_0$.

4. Evaluate approximately the ratio of mass of the Sun to that of Earth, using only the lengths of the year and of the lunar month (27.3 days), and the mean radii of Earths orbit (1.49×10^8 km) and of the Moons orbit (3.8×10^5 km)

5. A body moves in a spiral path of the form $r = c\theta^2$ (c a constant) under the influence of a central force with potential $V(r) = Ar^n + Br^m$ (A and B constants). What are the values of the exponents n and m?