Mechanics - PHY 6247

HW 2

READ: p. 34-64

HOMEWORK:

1. Show that the geodesics (paths of least distance) between two points on a spherical surface are great circles (i.e., circles whose centers lie at the center of the sphere).

2. a) Evaluate the action $S(\vec{r}(t))$ for a free particle in one dimension along the path: from (x_0, t_0) to (x, t) at constant velocity, and then, from (x, t) to (x_1, t_1) at a different constant velocity.
b) Consider S as a function of (x, t). Show that minimum action results when (x, t) is chosen so that the velocity from (x_0, t_0) to (x, t) is the same as that from (x, t) to (x_1, t_1) so that the full motion is at constant velocity.

3. Fermat's principle states that light travels from one point to another along the trajectory which makes the travel time a minimum.
a) Use Fermat's principle to derive the law for the reflection of light from a mirror. I.e. that the angle of incidence (as measured from the normal to the mirror) is equal to the angle of reflection.
b) Use Fermat's principle to derive Snell's law for the refraction of light passing from a medium in which the speed of light is c/n_0 to a medium where the speed of light is c/n_1 (c is the speed of light in vacuum and n is the refraction index of the medium):

$$n_0 \sin \phi_0 = n_1 \sin \phi_1$$

4. A particle moves vertically in the uniform gravitational field near the surface of the earth. The Lagrangian is $L = (1/2)m(dz/dt)^2 - mgz$. Suppose that at the time $t_0 = 0$ the particle is at $z = 0$ and at the time t_1 is at z_1. Pretend you don't know what the actual motion is. You might then guess that it can be adequately represented by the first three terms in a power series in t,

$$z(t) = z_0 + v_0 t + (1/2)at^2$$

where z_0 and v_0 are constants chosen such that $z(t)$ passes through the endpoints and a is an adjustable parameter. Evaluate the action S for this form of z(t) and note the dependence of $z(t)$ on a. For what value of a is S an extreme?

5. Find the plane curve $y = y(x)$ joining two points $(0, y_0)$ and (x_1, y_1) such that the area of the surface formed by rotating the curve about the x-axis is minimum?

6. As solid cylinder is rolling down an incline without slipping. Find the equations of motions and the force of constraint using Lagrange multipliers.