Mechanics - PHY 6247

HW 1

READ: p. 1-29

HOMEWORK:

1. Prove that the magnitude R of the center of mass vector (for any origin) is given by an expression of the form

$$M^2 R^2 = aM \sum_i m_i r_i^2 + b \sum_{ij} m_i m_j r_{ij}^2.$$

What are the coefficients a, b and what is r_{ij}?

2. Two wheels of radius a are mounted on the ends of a common axle of length b such that the wheels rotate independently. The whole combination rolls without slipping on a plane. We denote the coordinates of the point on the axle midway between the two wheels by x and y, and let θ be the angle between the axle and the x-axis, and ϕ, and ϕ' be the angles that measure how far each wheel has turned.
Show that there are two nonholonomic equations of constraint,

$$\cos\theta dx + \sin\theta dy = 0$$

$$\sin\theta dx - \cos\theta dy = (a/2)(d\phi + d\phi')$$

and one holonomic equation of constraint

$$\theta = C - (a/b)(\phi - \phi')$$

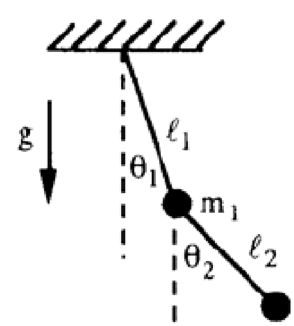
where C is a constant.

3. If L is a Lagrangian for a system, show that both L and

$$L' = L + \frac{dF(q_1, ..., q_n, t)}{dt}$$

lead to the same equations of motion. Here F is an arbitrary differentiable function of time.

4. Consider a double pendulum.
a) Find kinetic and potential energy of the pendulum.
b) Find the equations of motion.



5. The Lagrangian for two particles of masses m_1 and m_2 and coordinates $\vec{r}_1$ and $\vec{r}_2$ that interact via a potential $V(\vec{r}_1 - \vec{r}_2)$ is

$$L = (1/2)m_1|d\vec{r}_1/dt|^2 + (1/2)m_2|d\vec{r}_2/dt|^2 - V(\vec{r}_1 - \vec{r}_2)$$

a) Rewrite the Lagrangian in terms of the center of mass coordinates $\vec{R}$ and relative coordinates $\vec{r} = \vec{r}_1 - \vec{r}_2$.
b) Use Lagrange's equations to show that the center of mass and relative motions separate, that the center of mass moves with constant velocity and that the relative motion is that of a particle of reduced mass $\mu = (m_1 m_2)/(m_1 + m_2)$ in a potential $V(\vec{r})$.

6.A bead of mass m slides along a straight wire which makes an angle α and rotates with constant angular velocity ω.
a) Find the Lagrangian.
b) Find Lagrange's equations of motion.