Numerical Relativity - PHY 6938

HW 5

Hand in this homework.

READ: Chap 3., Appendix A.

PROBLEMS:

1. Let $\gamma_{ij} = \psi^4 \bar{\gamma}_{ij}$. Express the 3D Ricci scalar R (computed from γ^{ij}) in terms of $\bar{R}$ (computed from $\bar{\gamma}^{ij}$) and ψ.
(It would be useful if you also gave expressions for the Riemann and Ricci tensors.)

2. Consider a 3D symmetric tensor S^{ij}. Express $D_j(\psi^n S^{ij})$ in terms of $\bar{D}_j S^{ij}$, S^{ij} and ψ.
Hint: Recall that two covariant derivative operators differ by a tensor C^l_{ik}, e.g. $D_i q_k = \bar{D}_i q_k - C^l_{ik} q_l$. Read Wald's book if you do not believe this. We have $C^k_{ij} = \frac{1}{2}\gamma^{kl}(\bar{D}_j \gamma_{li} + \bar{D}_i \gamma_{lj} - \bar{D}_l \gamma_{ij})$

3. Recall $(LW)^{ij} := D^i W^j + D^j W^i - (2/3)\gamma^{ij} D_k W^k$. Compute $(LW)^{ij}$ in terms of ψ and $(\bar{L}W)^{ij}$

4. Consider a conformally flat 3-metric and let $\beta^i = (\Omega \times r)^i$, where $r^i = (x, y, z)$ is the position vector and Ω^i is constant. Compute $(L\beta)^{ij}$.

5. Use (A.12) to compute the ADM mass of standard puncture data with $\psi = \psi_{BL} + u$, where $\psi_{BL} = 1 + \sum_A \frac{m_A}{2r_A}$. Express your answer in terms of the m_A and an integral over u. Write the latter explicitly as an integral over θ and ϕ of terms containing $\partial_r u$.