Numerical Relativity - PHY 6938

HW 4

Hand in this homework.

READ: Chap 2.5-2.9

PROBLEMS:

1. Derive the Gauß-Codazzi and Codazzi-Mainardi equations.

2. Derive the relation

$$\gamma_a^c \gamma_b^d n^e n^f R_{cedf} = \pounds_n K_{ab} + K_{ac} K_b^c + (D_a D_b \alpha)/\alpha$$

from

$$(\nabla_a \nabla_b - \nabla_b \nabla_a) n_c = R_{abc}{}^d n_d.$$

3. In the BSSNOK formulation a new variable $\phi = \frac{1}{12} \ln \gamma$ is introduced (here γ is the determinant of the 3-metric). Its evolution equation is

$$\partial_t \phi = -\frac{1}{6} \alpha K + \beta^i \partial_i \phi + \frac{1}{6} \partial_i \beta^i.$$

However, within the now standard moving puncture formulation, one often replaces ϕ by $W = e^{-2\phi}$, since ϕ goes to infinity at punctures (which are used to simulate back holes). Derive the evolution equation for W.