Numerical Relativity - PHY 6938

HW 3

Hand in this homework.

READ: Chap 2.1-2.4

PROBLEMS:

1. In units where $G = c = 1$ we can express all physical quantities in units of meters.
a) Express the following in meters: 1s, 1kg, 1m/s, 1m/s^2
b) In units of $G = c = 1$ a particular black hole has a mass of m =1km. A particle circles around the black hole with an orbital angular velocity ω given by $m\omega = 0.05$. Determine ω in SI units.

2. Consider spatial hypersurfaces (given by $t = const$) with a normal vector n^a, where $n_a n^a = -1$. Define $\gamma_{ab} := g_{ab} + n_a n_b$, and introduce coordinates such that $x^0 = t$.
a) Compute all 4 components of n^a and n_a (the normal vector of a spatial slice) in terms of lapse and shift.
b) Compute the components of the 3-metric $\gamma^{0\mu}$ and $\gamma_{0\mu}$ in terms of lapse, shift and γ_{ij}
c) Show that γ^{ij} is the inverse of γ_{ij} (note here we use only spatial indices).

3. Consider a 3d hypersurface with a normal vector n^a, where $n_a n^a = -1$. Let us define $\gamma_{ab} := g_{ab} + n_a n_b$, $\gamma^b_a := g^b_c \gamma_{ac}$ and

$$K_{ab} := -\gamma^c_a \gamma^d_b \nabla_c n_d.$$

a) Show that K_{ab} is purely spatial.
b) Introduce a time coordinate t such that $t = 0$ is the 3d hypersurface. Use $n_a = k\nabla_a t$ (where k is a smooth function) to show that K_{ab} is symmetric.
c) Show that $K_{ab} = -\gamma^c_a \nabla_c n_b$
d) Express $\pounds_n \gamma_{ab}$ in terms of n_a and compute how it is related to K_{ab}.

4. Denote the normal vector and lapse by n^a and α. The tensor $S^a{}_b$ is purely spatial, i.e. $S^a{}_b n_a = 0 = S^a{}_b n^b$.
a) Compute $(\pounds_T S^a{}_b) n_a$ and $(\pounds_T S^a{}_b) n^b$, where $T^a = \alpha n^a$.
b) Can you show that $\pounds_n S^a{}_b$ is purely spatial as well?